Dear Family,

The next Unit in your child's mathematics class this year is ***Frogs, Fleas, and Painted Cubes: Quadratic Functions.*** In the Grade 6 unit *Variables and Patterns* and the Grade 7 unit *Moving Straight Ahead,* students were introduced to the basic concepts of algebra, and they examined linear models in detail. In the Grade 8 unit *Thinking With Mathematical Models,* they revisited the use of linear models and investigated several examples of inverse variations. They also explored exponential models in the *Growing, Growing, Growing* unit. In this Unit, the focus switches to a nonlinear polynomial relationship: the second-degree polynomial, or the *quadratic function.*

Unit Goals

Students will learn to recognize quadratic patterns of change in tables and graphs, and they will learn to write equations to represent those patterns. They will compare and contrast quadratic patterns of change with those of linear and exponential patterns of change, which they have already studied in depth.

Quadratic relationships are encountered in such fields as business, sports, engineering, and economics. We are dealing with quadratic relationships, for example, when we study how the height of a ball—or jumping flea—changes over time. A quadratic graph, called a *parabola,* has the shape of either a U or an upside-down U.

Homework and Conversations About the Mathematics

You can help with homework by asking questions such as the following:

- How can you recognize if the relationship among variables in a situation is a quadratic function?
- What equation would represent a quadratic relationship in the table, graph, or problem context relating the variables?
- How could you answer the questions of the situations by studying a table, graph, or equation of the quadratic function?

You can help your child with his or her work for this Unit in several ways:

- Talk with your child about the situations that are presented in the Unit.
- Search with your child for other situations that could be modeled by a quadratic equation, table, or graph.

Common Core State Standards

Students develop and use all of the Standards for Mathematical Practice throughout the curriculum. In this Unit, students make use of appropriate tools strategically as they use technology to explore important characteristics of quadratic relationships. They also look for and make use of structure as they write equivalent expressions. Quadratic functions and modeling is an Algebra I standard that is studied extensively.

A few important mathematical ideas that your child will learn in *Frogs, Fleas, and Painted Cubes* are given on the next page. As always, if you have any questions or concerns about this Unit or your child's progress in the class, please feel free to call.

Sincerely,

Important Concepts	Examples

Representing Quadratic Patterns of Change With Tables

In linear functions, the *first differences* of successive values are constant, indicating a constant rate of change. In quadratic functions, first differences are not constant, but *second differences* are. The first difference is the rate at which y is changing with respect to x. That is, the first difference gives the change in y-values between x and $x + 1$. The second difference indicates the rate at which *that rate* is changing. If the second differences are all the same, then the function is quadratic. Finding successive differences of polynomials relates to derivatives in calculus.

$y = 6(x - 2)^2$

x	y	First Differences	Second Differences
0	24		
		$6 - 24 = -18$	
1	6		$-6 - (-18) = 12$
		$0 - 6 = -6$	
2	0		$6 - (-6) = 12$
		$6 - 0 = 6$	
3	6		$18 - 6 = 12$
		$24 - 6 = 18$	
4	24		$30 - 18 = 12$
		$54 - 24 = 30$	
5	54		

The second differences are all 12, which indicates that the table represents a quadratic function.

Representing Quadratic Functions With Equations

Traditionally, quadratic functions are defined as functions that have equations fitting the form $y = ax^2 + bx + c$, in which a, b, and c are constants, and $a \neq 0$. This form of the equation is called the *expanded form*. The emphasis is on observing that the equations contain an independent variable raised to the second power. While this is a useful definition, it is also important to understand the *factored form* of such equations.

Quadratic equations arise from situations that have an underlying multiplicative structure, such as the area of rectangles. Thus, many quadratic equations can also be defined as functions whose y-value is equal to the product of two linear factors—the form $y = (ax + c)(bx + d)$, where $a \neq 0$ and $b \neq 0$. The power of this form is that it ties polynomials together as products of linear factors. The two factors for the factored form of a quadratic expression are also called *binomial expressions*, or *binomials*. It is an expression with two terms.

The area of the rectangle below can be thought of as the product of two linear expressions, the result of multiplying the width by the length, or as the sum of the area of the subparts of the rectangle.

	x	d
x	x^2	dx
c	cx	cd

$A = (x + c)(x + d)$ factored form

$A = x^2 + cx + dx + d$ expanded form

Representing Quadratic Patterns of Change With Graphs

The values in the equation affect the shape, orientation, and location of the quadratic graph, a parabolic curve.

If the coefficient of the x^2 term is positive, the curve opens upward and has a minimum point. If negative, the curve opens downward and has a maximum point.

The maximum or minimum point of a quadratic graph **(parabola)** is called the **vertex.** The vertex lies on the vertical *line of symmetry* that separates the parabola into halves that are mirror images. The vertex is located halfway between the **x-intercepts,** if the x-intercepts exist. The x-intercepts are mirror images of each other. The **y-intercept** is the point at which the parabola crosses the y-axis.
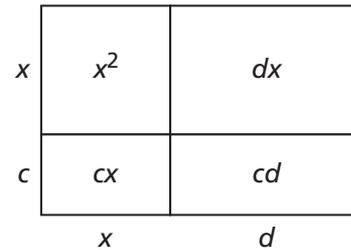

$y = -x^2 + 8x$

$y = x^2 - 8x + 16$